Answer the questions in the spaces provided on the question sheets. Supporting your answer with your work, a graph, and/or a short explanation is a good idea. Unsupported answers will not receive full credit.

Name: _____

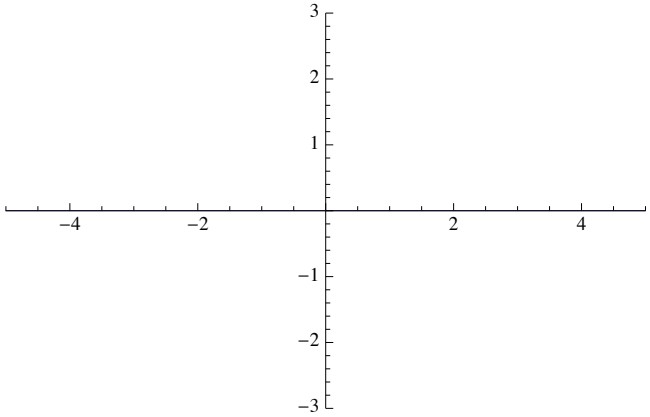
Section: _____

Question	Points	Score
1	16	
2	20	
3	31	
4	6	
5	10	
6	16	
7	25	
8	18	
9	19	
10	19	
11	20	
Total:	200	

1. You are given a line with slope -2 and y-intercept -1.

   (a) Give the equation of the line in either point-slope or slope-intercept form.

(4) (b) Graph the line L on these axes:



(4) (c) Give the equation of a line that is perpendicular to line L and goes through $(1, 2)$.

(4) (d) Give the equation of a line parallel to the original line L that passes through the origin.

(4) (e) Graph the lines you found in parts (b) and (c) on the axes above in part (a). Clearly label each line.

2. If it is possible, rewrite the following as polynomials and give the degree of the polynomial. If it is not possible to rewrite the function as a polynomial, explain why.

(5) (a) $\frac{1}{x^3}(x^6 - 8x^5 + x^4)$

(5) (b) $e^x + x^e$

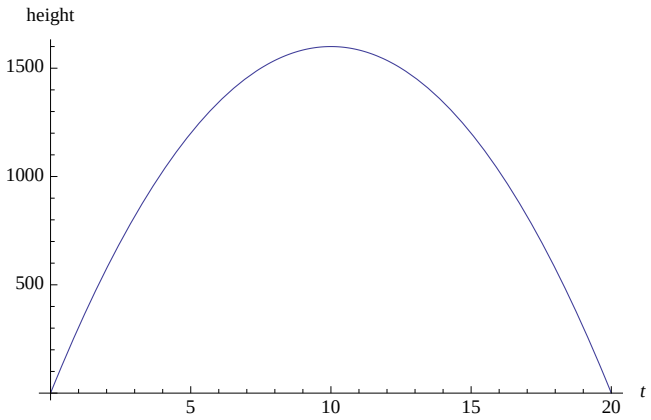
(5) (c) $x^{0.5}(x^{4.5} - 2x^{2.5})$

(5) (d) $\frac{x}{4x^5}$

3. A football is thrown by a player on our JV team at initial speed a little less than 30 mph. If the player throws the ball straight up in the air, we get a function for height $f(t)$ in feet as a function of t in seconds (I did the conversion from miles per hour to feet per second for you). The formula looks like

$$f(t) = -16t^2 + 320t$$

and the graph looks like



(3) (a) What is the input and what are its units?

(3) (b) What is the output and what are its units?

(3) (c) Is this concave down, concave up, or neither?

(4) (d) Use either the graph or algebra to find at what time the ball is at its maximum height. Explain how you found your answer.

(4) (e) Find the maximum height of the ball. Explain how you found your answer.

(5) (f) Write an expression for the average speed of the football between time $t = 0$ and time t seconds later. Give units for each part of your expression.

(5) (g) What is the average speed of the football between time $t = 0$ and $t = 5$? Use the fact that the football is at height zero at time zero, and height 1200 feet five seconds later. You do not have to simplify.

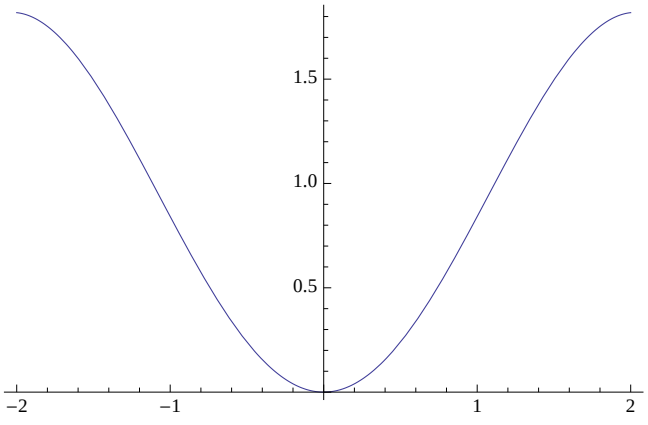
(4) (h) If Joey threw the ball at time zero (as described by the original equation, $f(t) = -16t^2 + 320t$) and then Cody threw the ball in the same way *45 seconds later* and from *one foot higher*, write the equation for the height $h(t)$ of the ball Cody threw.

The names and heights in this problem are not intended to accurately represent anything in real life...

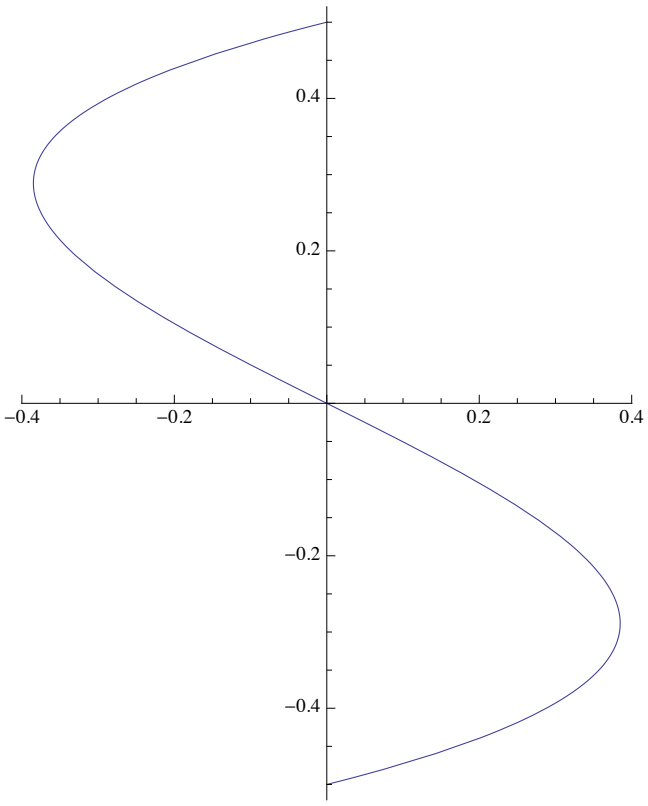
(6) 4. Use logical reasoning to decide if this is a function: *The number of students at St. Olaf graduating each year.* Explain your answer with a sentence.

5. Decide if the following graphs represent functions with input x. Explain with a short sentence why or why not, writing your explanation next to the graph.

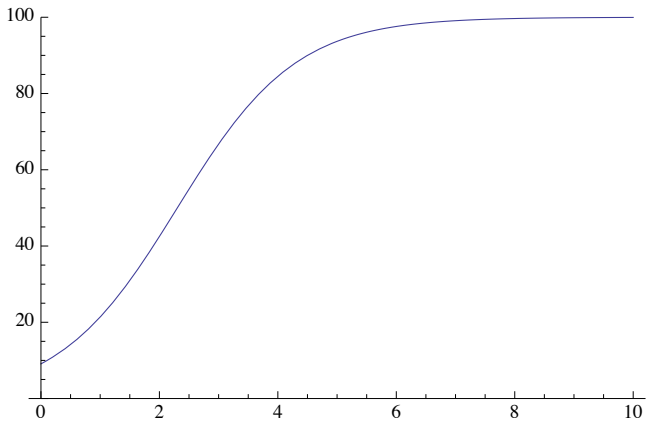
(5) (a)



(5) (b)



6. Betsy won the lottery and was able to buy nine alpacas to start her herd. Even though she won the lottery, she still has limited space and a limited food budget, so the herd grew logistically rather than exponentially. Here is a graph of her alpaca population $P(t)$ over ten years.



(3) (a) Using the graph and the situation described, what is the domain of the function $P(t)$?

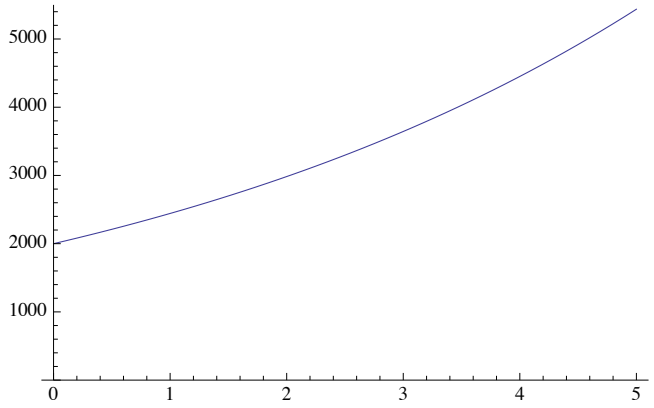
(3) (b) What is the range of the function $P(t)$?

(3) (c) For what values of t is the function concave up? Estimate.

(3) (d) For what values of t is the function concave down? Estimate.

(4) (e) What is $\lim_{t\to\infty} P(t)$?

7. Emma and Mariah encounter a terrible lab accident in one of their biology classes: another student dropped the petri dish containing the flesh-eating bacteria the class had been culturing. From the student's lab notebook they discover that there were 2000 bacteria at the time the dish was dropped. From properties of the bacteria they know that the *continuous* growth rate for the bacteria is 20% an hour.



(4) (a) Given the initial population of 2000 bacteria and the 20% continuous growth rate, write the formula for the population $P(t)$ at time t.

(3) (b) What is the domain of your function?

(3) (c) What is the range of your function?

(3) (d) Find $\lim_{x\to\infty} P(t)$.

(8) (e) What is the doubling time for the bacteria population? You will not be able to get a decimal approximation, but you can solve for it exactly.

(4) (f) Estimate the doubling time for the bacteria population using the graph above.

8. The equation for the temperature of a cup of green tea is

$$T(t) = Ae^{-kt} + 75.$$

A depends on starting temperature, and k is the continuous growth rate.

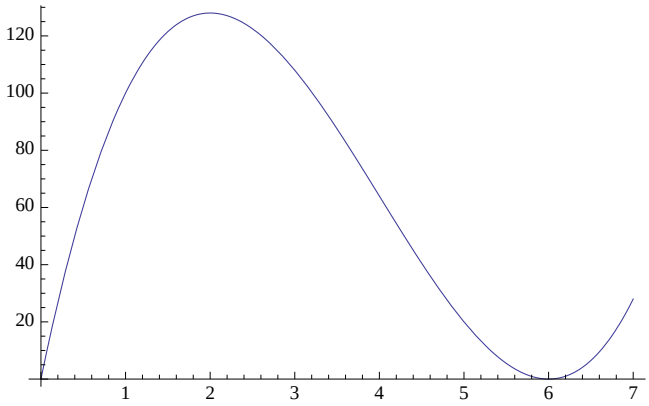
(4) (a) Is the function $T(t)$ decreasing or increasing? If you aren't sure, think about the physical situation.

(6) (b) The function given is the function Ae^{-kt} shifted up 75 units. Using what you know about exponentials and/or the physical situation, what is the limit for temperature as $t \to \infty$? Support your answer with a sketch of a graph or a sentence.

(4) (c) If your tea is at 175 degrees Fahrenheit when you pour the water over the leaves, you can find A using the equation $T(t) = Ae^{-kt} + 75$. Find A.

(4) (d) Black tea should be brewed using hotter water. If instead your water is at 210 degrees when you pour it over the tea leaves, what is A?

9. Nelson and Shiny have gone winter camping along with some other Oles. Their group got stuck in the Boundary Waters, and they need to improvise a brownie pan of maximum volume in order to feed the group and keep people from getting crabby. The formula for the volume of their brownie pan is $V(x) = x(12 - 2x)^2$: they have a 12 by 12 sheet of tin foil and they cut squares out of each corner and fold the sides up. The graph of this function is given below:



(4) (a) Given the physical situation, what range of x-values makes sense?

(6) (b) Estimate the x-value that gives the maximum volume of the brownie pan.

(6) (c) Give the roots of the function $V(x)$. What method did you use to find them? Explain.

(3) (d) What is the volume of the brownie pan at the roots of $V(x)$?

10. The campers had sent Aubrey to ski for help. She returns and gives them the following information: there is a resort nearby that will take them in if they can walk there. The campers know that in the first hour of walking they can go two miles, but each hour after that the number of miles they can walk will decrease by 20%.

(4) (a) In the first hour they walk two miles. How many miles do they walk in the second hour?

(6) (b) Write a formula for how many miles they can walk in t hours.

(4) (c) One group starts at time zero. The next group stays behind to pack up a bit and rest before heading out. They start three hours later. Write a formula for their mileage.

(5) (d) Is this an exponential or linear function? Explain your answer.

11. You are given two mystery functions: $f(x)$ and $g(x)$. They are a mystery because you only know the following information:

$$f(3) = 12, \qquad g(0) = 3, \qquad f(-8) = 0.$$

Use this information to answer the following questions, and if you think that there is not enough information to give an answer, tell me so!

(4) (a) $f(g(0)) =$

(4) (b) $g(f(-8)) =$

(4) (c) $f(g(-85)) =$

(4) (d) $f^{-1}(0) =$

(4) (e) $g(g^{-1}(4)) =$

I pledge on my honor that during this examination I have neither given nor received assistance, and that I have seen no dishonest work.

11. _____

I have intentionally not signed the pledge. (Check only if appropriate.)

11. _____